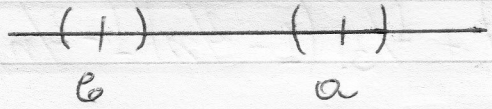
ДИС - Кирчев 15.10.12

Неравенства между границите

$$\left.\begin{array}{l} a_n \to a \\ b_n \to b \end{array}\right\} \Rightarrow \begin{array}{l} a_n \le b_n \\ \downarrow \quad\; \downarrow \\ a \le b \end{array}$$

Допускаме $a > b$

$b_n < a_n$



т.е. Невярно

Граничен преход

$$a_n \le c_n \le b_n$$

при $a = b \Rightarrow$

$$\left.\begin{array}{ll} b-\varepsilon < b_n < b+\varepsilon & n > N \\ a-\varepsilon < a_n < a+\varepsilon & n > N \end{array}\right\} \Rightarrow$$

$$a+\varepsilon < c_n < b+\varepsilon$$

$$a = b = c$$

$n > N \quad c-\varepsilon < c_n < c+\varepsilon$

Следователно c_n е на същата граница

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

при

$a_n < b_n$
$\downarrow \quad \downarrow$
$a \le b$
винаги
sign $\le$

при преход на границите

$b_1, b_2 \dots b_n \to e$

$$b_n = 2 + 1/2! + 1/3! + \dots 1/n!$$

$$2 < a_n < 1 + b_n < 3$$

$$\downarrow \qquad \downarrow$$

$$e \leq b \leq 3$$

$$0 < b_n < 3$$

$$e = (1 + 1/n)^n = a_n =$$

$$= 2 + 1/2!\left(1 - 1/n\right) + 1/3!\left(1 - 2/n\right)\left(1 - 3/n\right) + \dots$$

$$\dots 1/n!\left(1 - 1/n\right)\left(1 - 2/n\right) \dots \left(1 - \frac{n-1}{n}\right) \dots$$

(заместваме с) нека $m < n$

тогава $a_n > 2 + 1/2!\left(1 - 1/n\right) + \dots + 1/m!\left(1 - 1/n\right) \dots \left(1 - \frac{m-1}{n}\right)$

$$\downarrow$$

$$e \geq 2 + 1/2! + 1/3! \dots 1/m! = 1 + b_n$$

$$e \geq 1 + b_n \text{ или } e \geq 1 + b$$

$$e \geq 1 + b$$

но от условие имаме $1 + b_n \geq a_n$ или $1 + b \geq e$

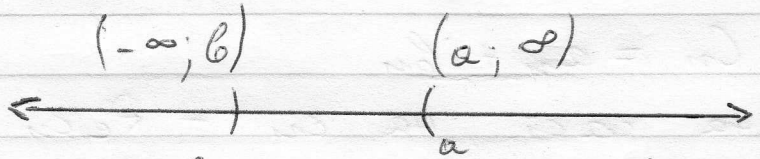
$$\left.\begin{array}{l} e \geq 1 + b \\ e \leq 1 + b \end{array}\right\} \Rightarrow \underline{e = 1 + b}$$

$$1 + 1 + 1/2! + 1/3! + \dots + 1/n! \longrightarrow e$$

$$\boxed{\sum_{n=1}^{\infty} 1/n! = b} \qquad \boxed{e = \sum_{n=0}^{\infty} 1/n!}$$

сума неперово число

Околност на безкрайно множество



$(-\infty; +\infty)$ интервал

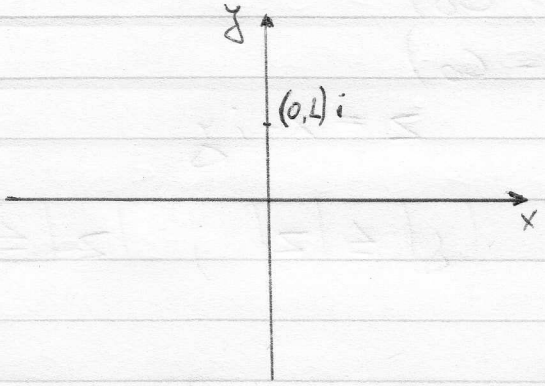
$a_1, a_2 \dots a_n \longrightarrow \infty$
$b_1, b_2 \dots b_n \longrightarrow -\infty$

(a, ∞) при $n > N$ $a_n \in (a; \infty)$

Ако

$$\left.\begin{array}{l} a_n \longrightarrow \infty \\ a_n \longrightarrow -\infty \end{array}\right\} \Rightarrow \frac{1}{a_n} \longrightarrow 0$$

$\mathbb{R}^2$



$$(x_1 + iy_1)(x_2 + iy_2) = x_1x_2 - y_1y_2 + i(x_1y_2 + x_2y_1)$$

$\mathbb{C}^2$

$c_1, c_2 \dots c_n \longrightarrow c$

$c_n = a_n + ib_n$

a_n - реална част на c_n - $Re\, c_n$

b_n - имагинерна част на c_n - $Im\, c_n$

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

$|z - z_0| < \varepsilon$ или

$$(x - x_0)^2 + (y - y_0)^2 < \varepsilon$$

$(x - x_0)^2 + (y - y_0)^2 = R^2$ $(x_0 y_0)$ - център на окръжност

$$|z - z_0| \leq R$$

$(a_n - a_0) < (c_n - c_0)$

$(b_n - b_0) < (c_n - c_0)$

$z = x + iy$

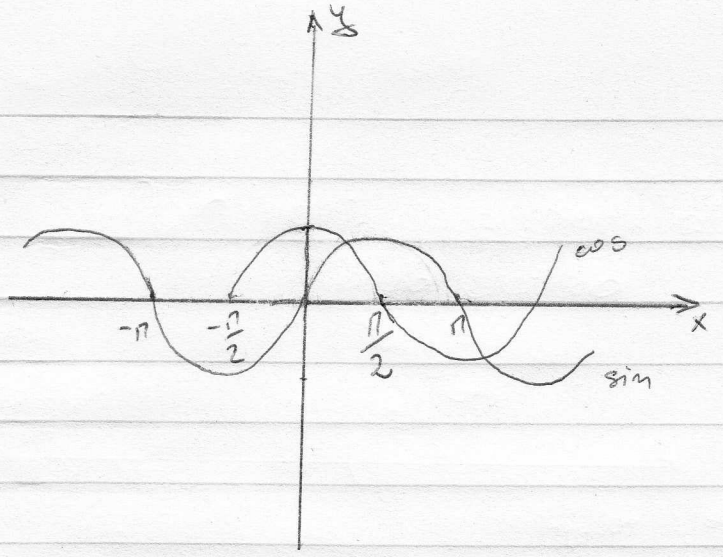
$|x| \leq |z|$; $|y| \leq |z|$; $|z| \leq |x| + i|y|$

$$|c_n - c_0| \leq |a_n - a_0| + i|b_n - b_0|$$

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} a_n + i \sum_{n=1}^{\infty} b_n$$

$\downarrow$ $\downarrow$ $\downarrow$

$S_n = S_n' + i S_n''$



$$tg = \frac{\sin x}{\cos x} \quad ; \quad ctg = \frac{\cos x}{\sin x}$$

$$\sin\left(\pi/2 - x\right) = \cos x \quad ;$$

$$\frac{1}{1 - tg^2 x} = \frac{1}{1 - \frac{\sin^2 x}{\cos^2 x}} = \cos^2 x$$

